

Study Plan :: 1st year

UC	Semester	ECTS	Remarks
Seminar	S1	9	compulsory
Research Project in Mathematics	S2	21	compulsory
Structural courses	S1	12	choose 2 UC
Optional courses	S1/S2	12	choose 9 ECTS for S1 and 3 ECTS for S2
Any 3rd cycle of studies	S2	6	choose 6 ECTS from any other 3rd cycle or from the Optional courses

The choice of all optional courses is subject to approval by the Scientific Committee of MAP-PDMA.

Syllabi

I - Structural courses :: Specialized Modules in Mathematics and Applications A

Tópicos Avançados de Álgebra, Lógica e Computação/ Advanced Topics in Algebra, Logic and Computation [S1 :: 56h :: 6 ECTS]

Each year are taught three of the following topics:

- Computer algebra: introduction to some computer algebra system; development of topics in computational number theory or in computational group theory.
- Automata, languages, and semigroups: regular languages; recognizability by finite state automata and by semi- groups; (option 1) varieties of semigroups and languages, Eilenberg's theorem; (option 2) Chomsky's hierarchy, decidability problems.
- Algebraic logic: elements of universal algebra; algebraization of classical, intuitionistic and modal logics; abstract algebraic logic.
- Category theory: universal properties; constructions in categories; natural transformations and adjunctions; monads.
- Proof theory: lambda-calculus; intuitionistic logic and Curry-Howard correspondence; proof systems.
- Matrix theory: elementary divisors and invariant factors, minimal polynomial; canonical forms of a matrix; nonnegative matrix, irreducibility and primitivity.

Tópicos Avançados de Análise e Otimização/ Advanced Topics in Analysis and Optimization [S1 :: 56h :: 6 ECTS]

Vector spaces: normed linear spaces; Banach spaces; separability; L_p -spaces. Hahn-Banach Theorem: Open Map- ping Theorem; dual spaces; reflexivity; weak and weak-* topologies. Hilbert spaces: the Projection Theorem; Stampacchia e Lax-Milgram Theorems; Riesz Representation Theorem. Application of the Hahn-Banach Theo- rem to minimum norm problems. Optimization of functionals: Gateaux and Frechet

derivatives; Euler-Lagrange equations; problems with constraints; convex-concave functionals; conjugate functionals; dual optimization problems. Global constrained optimization: Lagrange multipliers; sufficiency; sensitivity; duality. Local constrained optimization: Inverse function equality and inequality constraints. Application to optimal control: Pontryagin maximum principle.

Tópicos Avançados de Dinâmica e Geometria/Advanced Topics in Dynamics and Geometry [S1 :: 56h :: 6 ECTS]

Elementary geometry of submanifolds of $\mathbb{R}^n$: Parametrisations (or charts), tangent bundle, differentiable functions, submanifolds, transversality. Differential forms, de Rham cohomology. Basic concepts of dynamics in $\mathbb{R}^n$ (or in submanifolds of $\mathbb{R}^n$): Differential equations, stability of equilibria and of periodic solutions, hyperbolicity, stable and unstable manifolds, Poincaré map. Structural stability and bifurcations. The same concepts for the dynamics of recurrence relations.

Tópicos Avançados de Probabilidade e Estatística/Advanced Topics in Probability and Statistics [S1 :: 56h :: 6 ECTS]

Measurable spaces. Sequences of events. Measurable functions. Measures. Random variables, probability measures, fundamental properties. Probability spaces, types of probability laws. Integration in probability spaces and expectation. Inequalities. Some probability distributions. Independence and conditioning. Characteristic functions. Modes of convergence of sequences of random variables. Laws of large numbers. Central limit theorems. Multivariate distributions, conditional laws. Conditional expectation. Statistical models. Decision theory: risk functions, decision rules, criteria. Exponential families. Sufficiency. Point estimation, comparison of estimators, asymptotic properties, methods of estimation with emphasis on likelihood-based inference. Hypothesis tests and confidence sets.

II - Optional courses :: Specialized Modules in Mathematics and Applications B

Optional courses in Probability and Statistic

Análise Estatística Multivariada/Multivariate Statistical Analysis [S1 :: 42h :: 6 ECTS]

Multivariate random variables. Descriptive statistics. Distances. Generalized variances and algebraic/geometric aspects. Multivariate distributions: Multinormal, Wishart, Hotteling's T2 and Wilks. Inferences about mean vectors; testing for equality of covariance matrices; inferences about mean vectors from two populations. Confidence regions and comparison of multivariate Normal populations. MANOVA - "Multivariate Analysis of Variance". Principal Component Analysis; Factorial analysis. Classification (Cluster Analysis). Discriminant Analysis.

Complementos de Análise Matemática Aplicada/Complements of Applied Mathematical Analysis [S1 :: 21h :: 3 ECTS]

Introduction/revision of Stochastic Calculus. Model of the Financial Market. Introduction/revision of Numerical Methods and simulation of Stochastic Processes. Computation of European and American Options.

Complementos de Estatística/Complementary Statistics [S1 :: 21h :: 3 ECTS]

Tests of goodness of fit: Chi-square test and Kolmogorov-Smirnov test. Test of Independence. Signal test. Wilcoxon test. Location tests for two populations with paired samples. Mann-Whitney-Wilcoxon test. One-way analysis of variance. Methods for multiple comparisons. Kruskal-Wallis test. Two-way analysis of variance. Methods for multiple comparisons.

Sistemas com acontecimentos discretos/Discrete event system [S1 :: 21h :: 3 ECTS]

Introduction. Untimed Models of Discrete-Event Systems: languages and automata theory, analysis of untimed models. Timed Models of Discrete-Event Systems: timed state automata. Stochastic Timed Models for Discrete- Event Systems: introduction to stochastic processes, stochastic timed state automata, generalized semi-Markov process, Poisson process. Markov Chains: models, transition probability matrix, transient and steady state analysis for discrete-time Markov chains and continuous-time Markov. Controlled Markov Chains: Markov decision processes, approaches to the synthesis for Markov decision problems. Queueing Theory: queueing models, performance and dynamics of a queueing system, analysis of Markovian queueing systems, Markovian queueing networks, control of queueing systems. Discrete-Event Simulation: the event scheduling simulation scheme, the process-oriented simulation scheme, discrete-event simulation languages, output analysis. Sensitivity Analysis.

Processamento de Sinal e Séries Temporais/Signal Processing and Time Series Analysis [S1 :: 42h :: 6 ECTS]

Foundations of Statistical Signal Processing. Topics of probabilistic methods in signals and systems, signal joint analysis, modelling, spectral estimation and filtering; application to the analysis of time series in the frequency domain. Selected advanced topics of statistical signal processing, regarding novel methodologies and targeting both longstanding and emergent signal processing and time series applications, as: time-variant and wavelet analysis, adaptation, kernel based learning, independent component analysis, non-linear modeling, bayesian signal processing. State-space models (dynamic linear models) as a general time series modeling framework: ARMAX models, switching models, long-memory models, volatility models, longitudinal data analysis. Case study application and critical insight of the studied methods.

Sistemas Estocásticos/Stochastic Systems [S1 :: 42h :: 6 ECTS]

Module I: Stochastic processes. Markov processes. Markov chains in discrete and continuous time. Wiener process. Stochastic Calculus. Wiener integral. Itô processes and Itô formula. Stochastic differential equations. Linear stochastic differential equations.

Module II: Simulation of stochastic systems in discrete and continuous time. Queueing models. Structure and classification. Key performance measures. Little's Law and fundamental relationships. Process of birth and death. Analysis of systems with one line. Main features. Modeling of systems for which the service time has arbitrary distribution. Formula of Pollaczek-Khinchin. Characterization of the distribution of time spent in the system by using embedded Markov chains. Queues of M/G/1 type with pauses of the server.

Module III: Discretization methods of EDEs. Simulation of time series. Simulation of stochastic differential systems.

Tópicos em Genética Estatística/Topics in Statistical Genetics [S1 :: 21h :: 3 ECTS]

Mendelian inheritance. Modes of transmission and risks calculation. Epidemiologic principles. Measures of disease frequency and association. Familial risk of disease. Segregation analysis. Linkage analysis: recombination fraction, genetic distance, haplotype. Lodscore methods. Population genetics: Hardy-Weinberg equilibrium. Linkage disequilibrium. Association studies.

Análise de Dados Espaciais/Spatial data analysis [S2 :: 21h :: 3 ECTS]

Identify spatially continuous data (geostatistics). Descriptive analysis of spatially distributed data. Types of stationarity (strong, weak or intrinsic) and isotropy. Estimation of spatial dependence (variogram and co-variogram). Spatial prediction using simple, ordinary or universal kriging. The gaussian geostatistical model and the generalized linear spatial model. Modelling lattice data: spatial association measures; the conditional and the simultaneous auto-regressive models, CAR and SAR (e.g. disease mapping). Brief introduction to spatio-temporal data.

Bioestatística/Biostatistics [S2 :: 42h :: 6 ECTS]

Biological data. Population and sample. Regression models. Linearization of the models. Estimation of regression function. Inferences in regression analysis. Adequacy of the regression model. Coefficient determination. Correlation coefficient. One-way analysis of variance: comparing several means, the analysis of variance F test, conditions for ANOVA, pairwise multiple comparisons. Two-way analysis of variance: conditions, main effects, and interaction. Multiple Regression Model. Multiple Regression Models. Partial coefficient of determination and adjusted coefficient of determination. Properties of least squares estimators. Estimation of variance. Confidence intervals and hypothesis testing in multiple regression. Evaluating the appropriateness of the model.

Extração de Conhecimentos em Fluxos Contínuos de Dados/Data Stream Mining [S2 :: 21h :: 3 ECTS]

Spatio-temporal data analysis. Data streams: main algorithms for classification, clustering, novelty and change detection. Case studies. Social Network Analysis.

Métodos Estatísticos em Epidemiologia/Statistical Methods in Epidemiology [S2 :: 21h :: 3 ECTS]

Epidemiological research: objectives and methods. Typology and design of epidemiologic research. Measures of disease frequency, measures of association and measures of potential impact. Validity of epidemiological research: selection bias, information bias and confounding. Principles and procedures of epidemiologic analysis: options for control of extraneous factors, interaction, effect modification and synergism. Modeling: theoretical considerations and analysis strategy.

Modelos Lineares Generalizados/Generalized Linear Models [S2 :: 21h :: 3 ECTS]

Review of linear models. Introduction to generalized linear models. Estimation of the model parameters, hypothesis testing and confidence intervals. Selection and validation of models. Regression models for binary data. Regression models for count data. Regression models for asymmetric models.

Optional courses in Dynamic and Geometry

Equações diferenciais com simetria/Differential equations with symmetry [S2 :: 42h :: 6 ECTS]

Qualitative study of differential and difference equations, parameter dependence, bifurcation and stability. Study of symmetry breaking in steady-state and Hopf bifurcations. The aim is to show how symmetries may be used systematically to analyze, predict, and understand dynamic behavior. Tools to be used are Lie groups; representations and actions; invariant and equivariant maps, Hilbert-Weyl theorem.

Métodos Avançados em Teoria da Relatividade/Advanced Methods in Relativity Theory [S2 :: 42h :: 6 ECTS]

Review of the main results and geometrical structure of Special Relativity. Foundations and main results of Differential Geometry. Metric connexion and covariant derivative. Manifold curvature and associated tensors. Parallel transport. Geodesic equations. The relation between curvature and gravitation. The Equivalence Principle. The Einstein field equations. Einstein Equations solutions. The perihelion shift effect. Light deflection in the presence of strong gravitational fields. Interior and exterior Schwarzschild solutions. Cosmological solutions. Gravitational waves.

Teoria de Bifurcação/Bifurcation Theory [S2 :: 42h :: 6 ECTS]

Introduction to the study of the alteration of the qualitative behavior of dynamical systems with parameters. These alterations include, for instance, the appearing and disappearing of equilibrium states, changes in the periodic behavior of solutions, changes on the stability or lead to a chaotic behavior.

Optional courses in Numerical Analysis and Computational Methods

Álgebra Linear Numérica/Numerical Linear Algebra [S1 :: 42h :: 6 ECTS]

Errors in matrix computations. Error analysis in basic matrix computations. Backward error analysis. Conditioning of linear systems and of eigenvalues and eigenvectors. Block algorithms. Permutation matrices, Gaussian elimination matrices, Givens' matrices and Householder's matrices. LU, QR and Choleski decompositions. Block versions of the decompositions. The routines from BLAS. Systems of linear equations Sparse matrices storage. Iterative methods for sparse systems. The basic methods of Jacobi, Gauss-Seidel and SOR. Krylov-type methods. Preconditioning techniques. Eigenvalues and eigenvectors. Orthogonal similarity reductions to Hessenberg and tridiagonal forms. The QR method. The bisection method for symmetric tridiagonal matrices. The power method and inverse iteration. Lanczos' and Arnoldi methods Singular values. The singular value decomposition (SVD). The Golub-Kahan and Lanczos methods for the reduction to bidiagonal form. QR-type and bisection methods for the bidiagonal SVD problem.

Métodos Espectrais Numéricos I/Numerical Spectral Methods I [S1 :: 21h :: 3 ECTS]

Introduction to spectral methods. Weighted residual method. Approximation to the function. Differential equations: Galerkin method, Tau method and collocation method. Periodic problems. Fourier transforms. Truncated Fourier transform. Discrete truncated Fourier transform (FFT). Elimination of aliasing. Multi-dimensional Fourier transform. Differential equations with constant and variable coefficients. Introduction to evolution equations. Heat equation. Explicit and implicit discretization. Convergence, consistency and stability. Numerical methods for the advection-diffusion equation. Spectral and pseudo-spectral methods. Presentation and resolution of many examples in FORTRAN 77, MATLAB or Python.

Tópicos de Análise Numérica/Topics in Numerical Analysis [S1 :: 21h :: 3 ECTS]

Errors and stability. Floating point systems. Errors and their propagation. Stability and conditioning. Interpolation and Approximation. The general problem of interpolation. Lagrange and Hermite polynomial interpolation. Splines. Trigonometric interpolation. Fourier series. The problem of best approximation. Best approximation in inner product spaces. Least square approximation. Minimax approximation. Quadrature Newton-Cotes rules (simple, composite and adaptive). Peano kernel theorem. Gauss-Christoffel rules. Classical orthogonal polynomials. Gaussian quadrature. Extrapolation methods. Linear Systems Direct methods for solving linear systems: triangular systems; Gaussian elimination; LU decomposition; Cholesky decomposition. Conditioning. Nonlinear equations. Iterative methods for solving scalar nonlinear equations. Fixed point iteration. Secant method. Newton's method. Multiple roots. Nonlinear systems: Newton's method and some variants.

Métodos Numéricos para Equações Diferenciais/Numerical Methods for Differential Equations [S2 :: 21h :: 3 ECTS]

Initial value problems. Existence theorem. One-step numerical methods. Euler and Runge-Kutta methods: stability, consistency and convergence analysis. Regions of absolute stability. Multi-step methods. Study of the stability and convergence. Taylor methods. Predictor-corrector methods. Boundary value problems. Finite difference methods.

Optional courses in Control and Optimization

Códigos e Sistemas/Codes and Systems [S1:: 21h :: 3 ECTS]

Introduction to the theory of polynomial matrices: unimodular matrices, right prime matrices and column reduced matrices. Convolutional code over a finite field. Encoders of a convolutional code: definition and properties. Realization of an encoder through a state space model: definition and an algorithm to construct a minimal realization of an encoder. Structure of the minimal encoders.

Controlo Não Linear/Nonlinear Control [S1 :: 21h :: 3 ECTS]

Introduction to nonlinear systems. Mathematical preliminaries. Fundamental properties of nonlinear system. Lyapunov stability. Input-Output stability. Advanced stability. Stability of perturbed systems. Nonlinear design tools.

Cálculo das Variações/Calculus of Variations [S2 :: 21h :: 3 ECTS]

We address the Calculus of Variations from an integrated point of view. Example of topics to be studied: equivalence between different formulations of the problems; existence of minimizers; necessary optimality conditions; sufficient conditions for optimality; applications.

Otimização Estratégica em Economia/Strategic optimization in Economy [S2 :: 42h :: 6 ECTS]

General equilibria theory. Game theory. Applications to energy and climate change.

Otimização e Desenho de Redes/Optimization and Network Design [S2 :: 42h :: 6 ECTS]

Models and algorithms used to solve (combinatorial) optimization problems that appear in network (telecommunications, transport, etc.) design problems. Basic concepts: flows, cuts, paths, etc. Discussion of formulations. Solution techniques: relaxations, cutting planes and heuristics. Network design with high level of service: availability, reliability, survivability.

Teoria de Otimização/Optimization Theory [S2 :: 21h :: 3 ECTS]

Basic concepts: Elements of convex analysis, convex sets and convex functions, settings, properties and examples. Classification of problems: with and without constraints, convex and non-convex problems, specific problems. Special cases of constraints in the form of equality and inequality, infinite constraints. Unconstrained optimization: optimality conditions, the notion of optimization algorithm and the properties of algorithms. Optimization methods without constraints and without calculus of derivatives. Constrained optimization: classification of problems, optimality conditions I and II order. Classification of optimization methods with constraints: methods of convex and linear programming, barrier functions and penalty interior point methods, methods based on Lagrange multipliers. Problems with linear constraints. Introduction to Data Mining: Optimization methods in solving the problems of classification and clustering.

Optional courses in Analysis

Equações com Derivadas Parciais/Partial Differential Equations [S2 :: 42h :: 6 ECTS]

Holder spaces. Distributions. Sobolev spaces. Traces and imbeddings. Extension theorems. Second order partial differential elliptic equations: existence of weak solutions and regularity. Maximum principle and Harnack inequality. Schauder estimates. Second order parabolic equations: existence of weak solutions and regularity. Maximum principle and Harnack inequality. Monotone parabolic equations.

Equações Diferenciais/Differential Equations [S2 :: 21h :: 3 ECTS]

General Theory: vector fields, flow, rectification, existence and uniqueness of solution, first integrals, phase curves, conservative systems. Linear Equations: fundamental solution and stability. Non-Linear Equations: stability, local and global analysis.

Inequações Variacionais e Quasi-Variacionais/Variational and quasi-variational inequalities [S2 :: 21h :: 3 ECTS]

Projection theorem and Lions-Stampacchia theorem. The obstacle problem and Lewy-Stampacchia inequalities. Mosco convergence of convex sets. Fixed point theorems. Penalization methods. Quasi-variational inequalities.

Optional courses in Algebra, Logic and Computation

Teorias de Galois/Galois Theories [S1 :: 42h :: 6 ECTS]

Classical Galois theory, Galois theory of Grothendieck, Infinitary Galois theory, Categorical Galois theory of commutative rings, Categorical Galois theorem and factorization systems, Covering maps.

Reticulados e Estruturas Algébricas Ordenadas/Lattices and Ordered Algebraic Structures [S2 :: 21h :: 3 ECTS]

Ordered Semigroups: the ordered monoid of residuated mappings on an ordered set; naturally ordered semigroups; ordered regular semigroups; ordered inverse semigroups; representation of some classes of ordered semigroups. Lattices: distributive lattices; complemented lattices, Boolean algebras; pseudocomplemented lattices, Stone algebras.

Teoria Algébrica das Inversas Generalizadas/Algebraic theory of generalized inverses [S2 :: 21h :: 3 ECTS]

The concept of generalized inverse, families of generalized inverses, generalized invertibility of matrices over certain algebraic structures, in rings and some of its subclasses, and in C^* -algebras.

Elementos da Teoria de Semigrupos/Elements of Semigroup Theory [S2 :: 21h :: 3 ECTS]

Fundamental aspects of semigroup theory. Regular semigroups, inverse semigroups and completely simple semigroups.