Minho :: Aveiro :: Porto
PhD Program in Applied Mathematics
2023/2024

Study Plan :: 1st year

UC	Semester	ECTS	Remarks
Seminar	S1	9	compulsory
Research Project in Mathematics	S2	21	compulsory
Structural courses	S1	12	choose 2 UC
Optional courses	S1/S2	12	choose 9 ECTS for S1 and 3 ECTS for S2
Any 3rd cycle of studies	S2	6	choose 6 ECTS from any other 3rd cycle or from the Optional courses

The choice of all optional courses is subject to approval by the Scientific Committee of MAP-PDMA.

Seminário/Seminar [S1 :: 9 ECTS]

This course is aimed at developing communication skills as well as a general knowledge of Mathematics and its applications. It includes a cycle of research seminars. In the end, the students must give a seminar on one of the topics presented in the cycle seminars. It will help the students to prepare for research activities and to strengthen their capacity to carry out the research work at the adequate level required for a PhD degree.

– The students must attend at least 8 seminars.
– The students must write a summary with discussion for 5 of the 10 seminars.
– These summaries must not exceed 1 page A4 (equivalent to times new roman 11pt letter size) each and must be submitted within 10 days after the seminar.
– Each student must choose one of the research themes proposed and prepare a 40m talk. This will be accomplished under the supervision of the faculty who proposed the research theme.
– Final grade is given in points out of 20: if the talk is not up to the standard, the student is given a second chance to resubmit the talk within 2 weeks.

Research Project in Mathematics/Research Project in Mathematics [S2 :: 21 ECTS]

During the first year, each student chooses a research theme and a thesis supervisor and writes a thesis proposal (PIM). At the end of the second semester, each student will present the thesis proposal (PIM) to a committee for review and approval. The student has 25 minutes for presentation, followed by a period of 15 minutes for discussion.

[Detailed information will be send to the students]

Syllabi

I - Structural courses :: *Specialized Modules in Mathematics and Applications A*

Tópicos Avançados de Álgebra, Lógica e Computação
Advanced Topics in Algebra, Logic and Computation [S1 :: 56h :: 6 ECTS]

Each year are taught three of the following topics:
– **Computer algebra: introduction to some computer algebra system; development of topics in computational number theory or in computational group theory**;
– **Automata, languages, and semigroups: regular languages; recognizability by finite state automata and by semigroups**; (option 1) varieties of semigroups and languages, Eilenberg's theorem; (option 2) **Chomsky's hierarchy, decidability problems**;
– Algebraic logic: elements of universal algebra; algebraization of classical, intuitionistic and modal logics; abstract algebraic logic.
– Category theory: universal properties; constructions in categories; natural transformations and adjunctions; monads.
– Proof theory: lambda-calculus; intuitionistic logic and Curry-Howard correspondence; proof systems.
– **Matrix theory: elementary divisors and invariant factors, minimal polynomial; canonical forms of a matrix; nonnegative matrix, irreducibility and primitivity.**

Tópicos Avançados de Análise e Optimização
Advanced Topics in Analysis and Optimization [S1 :: 56h :: 6 ECTS]

Vector spaces: normed linear spaces; Banach spaces; separability; Lp-spaces. Hahn-Banach Theorem: Open Mapping Theorem; dual spaces; reflexivity; weak and weak-* topologies. Hilbert spaces: the Projection Theorem; Stampacchia e Lax-Milgram Theorems; Riesz Representation Theorem. Application of the Hahn-Banach Theorem to minimum norm problems. Optimization of functionals: Gateaux and Frechet derivatives; Euler-Lagrange equations; problems with constraints; convex-concave functionals; conjugate functionals; dual optimization problems. Global constrained optimization: Lagrange multipliers; sufficiency; sensitivity; duality. Local constrained optimization: Inverse function equality and inequality constraints. Application to optimal control: Pontryagin maximum principle.

Tópicos Avançados de Dinâmica e Geometria
Advanced Topics in Dynamics and Geometry [S1 :: 56h :: 6 ECTS]

Elementary geometry of submanifolds of $\mathbb{R}^n$: Parametrisations (or charts), tangent bundle, differentiable functions, submanifolds, transversality. Differential forms, de Rham cohomology. Basic concepts of dynamics in $\mathbb{R}^n$ (or in submanifolds of $\mathbb{R}^n$): Differential equations, stability of equilibria and of periodic solutions, hyperbolicity, stable and unstable manifolds, Poincaré map. Structural stability and bifurcations. The same concepts for the dynamics of recurrence relations.

Tópicos Avançados de Probabilidade e Estatística
Advanced Topics in Probability and Statistics [S1 :: 56h :: 6 ECTS]

Measurable spaces. Sequences of events. Measurable functions. Measures. Random variables, probability measures, fundamental properties. Probability spaces, types of probability laws. Integration in probability spaces and expectation. Inequalities. Some probability distributions. Independence and conditioning. Characteristic functions. Modes of convergence of sequences of random variables. Laws of large numbers. Central limit theorems. Multivariate distributions, conditional laws. Conditional expectation. Statistical models. Decision theory: risk functions, decision rules, criteria. Exponential families. Sufficiency. Point estimation, comparison of estimators, asymptotic properties, methods of estimation with emphasis on likelihood based inference. Hypothesis tests and confidence sets.

II - Optional courses :: *Specialized Modules in Mathematics and Applications B*

:: *Algebra, Logic and Computation*
:: *Analysis*
:: *Control and Optimization*
:: *Dynamic and Geometry*
:: *Numerical Analysis and Computational Methods*
:: *Probability and Statistics*

Optional courses in Algebra, Logic and Computation

Teoria Algébrica das Inversas Generalizadas/Algebraic theory of generalized inverses [S2 :: 21h :: 3 ECTS]

The concept of generalized inverse, families of generalized inverses, generalized invertibility of matrices over certain algebraic structures, in rings and some of its subclasses, and in C^*-algebras

Teoria de Tipos/Type Theory [S1 :: 42h :: 6 ECTS]

Simple types, polymorphic types, dependent types: meta-theory, propositions-as types principle, relationship with constructive logics. Optional topics: inductive definitions and termination of recursive functions in Type Theory; proof assistants; type systems based on sequent calculus or Classical Logic; subtyping; etc.

Teorias de Galois/Galois Theories [S1 :: 42h :: 6 ECTS]

Classical Galois theory, Galois theory of Grothendieck, Infinitary Galois theory, Categorical Galois theory of commutative rings, Categorical Galois theorem and factorization systems, Covering maps.

Optional courses in Analysis

Tópicos de Teoria Cinética/Topics of Kinetic Theory [S2 :: 42h :: 6 ECTS]

Boltzmann equation (collisional dynamics, collisional operator, properties, conservation laws, equilibrium, H-theorem). Linearized Boltzmann equation. Existenceand uniqueness of solution. Hydrodinamic limit. Boltzmann equation for mixtures, for chemically reactive mixtures and for polyatomic gases. Applications to mathematical biology and combustion. Modelling, numerical analysis and simulation of these problems.

Tópicos de Turbulência/Topics in Turbulence [S2 :: 21h :: 3 ECTS]

Navier-Stokes equations. Vorticity dynamics. Statistical description of turbulent flows. Coherent structures. Isotropic turbulence (spectral formalism, Kolmogorov theory). Anisotropic turbulence (SO(3) formalism). Discussion on the existence and uniqueness of solutions of the Navier-Stokes equations. Eddy viscosities. Point vortices. Passive scalars. Direct numerical simulation.

Polinómios Ortogonais e Aplicações/Orthogonal Polynomials and Applications [S2 :: 21h :: 3 ECTS]

Elements of the General Theory of Orthogonal Polynomials. Boundary Value Problems in the Theory of Analytic Functions. Riemann-Hilbert Method for Orthogonal Polynomials on [-1,1]. Non-Linear Steepest Descent Method. Asymptotic results for Orthogonal Polynomials on [-1,1].

Optional courses in Control and Optimization

Otimização e Desenho de Redes/Optimization and Network Design [S1 :: 42h :: 6 ECTS]

Models and algorithms used to solve (combinatorial) optimization problems that appear in network (telecommunications, transport, etc.) design problems. Basic concepts: flows, cuts, paths, etc. Discussion of formulations. Solution techniques: relaxations, cutting planes and heuristics. Network design with high level of service: availability, reliability, survivability.

Optimização Estratégica em Economia/Strategic optimization in Economy [S2 :: 42h :: 6 ECTS]

General equilibria theory. Game theory. Applications to energy and climate change.

Cálculo das Variações/Calculus of Variations [S2 :: 21h :: 3 ECTS]

We address the Calculus of Variations from an integrated point of view. Example of topics to be studied: equivalence between different formulations of the problems; existence of minimizers; necessary optimality conditions; sufficient conditions for optimality; applications.

Controlo de Sistemas Incertos e Estruturados/Control of Uncertain and Structured Systems [S2 :: 42h :: 6 ECTS]

Introduction. Recursie Estimation. Adaptive Control. Distributed Systems. Centralized and Decentralized Controlo of Distributed Systems. Distribued Estimation. Consensus Generation Methods. Data-driven Control.

Controlo de Sistemas Lineares/Control of Linear Systems [S2 :: 42h :: 6 ECTS]

Linear state space models of linear multivariable systems, solution in the frequency domain and in the time domain (transfer function and impulse response). Linear state space models as linearization of nonlinear systems. Structural properties. Realization theory. System interconnection. Control problems with state measurement: Pole placement and stabilization by static state feedback; Tracking and disturbance rejection; Linear quadratic control problems.

Controlo de Sistemas Dinâmicos Não-Lineares/Control of Non-linear Dynamical Systems [S2 :: 42h :: 6 ECTS]

Introduction. Invariance. Attainability. Stability. Robustness.

Sistemas Discretos e Híbridos/Discrete and Hybrid Systems [S2 :: 21h :: 3 ECTS]

Introduction. Systems with untimed and timed discrete events. Systems with stochastic timed discrete events. Markov Chains: Controlled Markov Chains; Markov Decision Processes; Markov Decision Problem Synthesis; Application to Queue Theory. Hybrid Systems Modeling: State Machines and Composition of these with dynamic systems;mConcepts and basic results of linear, nonlinear and switched systems; Articulation of event-driven and time-driven dynamics. Generic Formal Model. Simulation of hybrid systems using MATLAB and StateFlow 8. Design of Controllers: Supervisors; Reachability and Safety Issues; Specification; Synthesis Techniques.

Optional courses in Dynamic and Geometry

Alterações Climáticas e Energia/Climate change and energy [S2 :: 42h :: 6 ECTS]

Dynamics and economical models in Game Theory, General equilibrium theory and Mathematical Finance, applied to climate change and energy models.

Dinâmica em Modelos Biológicos/Biological Dynamics [S2 :: 42h :: 6 ECTS]

Dynamic models in Ecology, Epidemiology, Immunology and in Biological systems. Introduction to Dynamical equilibria, stability, Lyapunov functions, bifurcations, topological chaotic dynamics. Stochastic stability, quasistationary equilibria. Nash equilibria, Stochastically stable equilibrium, evolutionarily stable state, evolutionary game theory. SIS, SIR, SIRI models: thresholds, bifurcations, mean field approximation, pair approximation, quasistationary equilibria, vaccination, game theoretical effects in vaccination. T-cells and regulatory T cells dynamics modeling: immune deceases, bifurcation thresholds, limit cycles, data analysis. Systems biology and dynamic analysis. Bioenergy models.

Métodos Avançados em Relatividade Geral/Advanced Methods in General Relativity [S2 :: 42h :: 6 ECTS]

Recap on Special Relativity. Motivation for General Relativity. Notions of Differential Geometry (tensors, linear connection, covariant derivative, parallel transport, curvature, geodesics). The field equations of General Relativity. The classical tests. Black holes. Gravitational waves. Additional topic.

Optional courses in Probability and Statistics

Análise de Dados Longitudinais/Longitudinal Data Analysis [S1 :: 21h :: 3 ECTS]

Linear Mixed Models for continuous data. Marginal models for discrete data. Correlation structures for correlated data. Variograma on longitudinal data. Diagnostics for longitudinal models. Method of maximum likelihood and restricted maximum likelihood. "Missing" data and processes of "Missing" data. Joint modeling of longitudinal and survival data.

Sistemas Estocásticos e Aplicações/Stochastic Systems and Applications [S1 :: 42h :: 6 ECTS]

Main classes of stochastic processes (Poison, Markov, times series, etc). Wiener processes and the Wiener integral (properties and Ito formula). Stochastic differential equations (OU, CIR, Black-Scholes and others). Stochastic systems discretization (Euler-Maruyama and superior order schemes). Simulation of stochastic processes (Poisson, Markov chains, solutions of SDEs, queues, time series). Estimation of parameters and statistical tests. Applications in Physics, Engineering and Biology (models of statistic mechanics, of energy and population growth). Financial Mathematics (financial markets models, risk measures; financial derivatives, evaluation of American and European options).

Processamento de Sinal e Séries Temporais/Signal Processing and Time Series Analysis [S1 :: 42h :: 6 ECTS]

Foundations of Statistical Signal Processing. Topics of probabilistic methods in signals and systems, signal joint analysis, modelling, spectral estimation and filtering; application to the analysis of time series in the frequency domain. Selected advanced topics of statistical signal processing, regarding novel methodologies and targeting both longstanding and emergent signal processing and time series applications, as: time-variant and wavelet analysis, adaptation, kernel based learning, independent component analysis, non-linear modeling, bayesian signal processing. State-space models (dynamic linear models) as a general time series modeling framework: ARMAX models, switching models, long-memory models, volatility models, longitudinal data analysis. Case study application and critical insight of the studied methods.

Modelos Lineares Generalizados/Generalized Linear Models [S1 :: 21h :: 3 ECTS]

Review of linear models. Introduction to generalized linear models. Estimation of the model parameters, hypothesis testing and confidence intervals. Selection and validation of models. Regression models for binary data. Regression models for count data. Regression models for asymmetric models.

Análise Multivariada e Aprendizagem Estatística/Multivariate Analysis and Statistical Learning [S2 :: 42h :: 6 ECTS]

Multivariate normal model: distributional properties, estimation of covariance and correlation matrix (large and small sample). Multivariate regression. Dimension reduction and latent variable models: Principal Components Analysis (PCA), Canonical Correlation Analysis (CCA), Factor Analysis (FA). Unsupervised learning / clustering: algorithmic approaches (e.g. K-means, hierarchical clustering). Supervised learning / classification: Linear and Quadratic Discriminant Analysis (LDA and QDA). Validation Approaches.

Análise de Dados Espaciais/Spatial data analysis [S2 :: 21h :: 3 ECTS]

Identify spatially continuous data (geostatistics). Descriptive analysis of spatially distributed data. Types of stationarity (strong, weak or intrisic) and isotropy. Estimation of spatial dependence (variograma and covariogram). Spatial prediction using simple, ordinary or universal kriging. The gaussian geostatistical model and the generalized linear spatial model. Modelling lattice data: spatial association measures; the conditional and the simultaneous auto-regressive models, CAR and SAR (e.g. disease mapping). Brief introduction to spatio-temporal data.

Bioestatística/Biostatistics [S2 :: 42h :: 6 ECTS]

Biological data. Population and sample. Regression models. Linearization of the models. Estimation of regression function. Inferences in regression analysis. Adequacy of the regression model. Coefficient determination. Correlation coefficient. One-way analysis of variance: comparing several means, the analysis of variance F test, conditions for ANOVA, pairwise multiple comparisons. Two-way analysis of variance: conditions, main effects, and interaction. Multiple Regression Model. Multiple Regression Models. Partial coefficient of determination and adjusted coefficient of determination. Properties of least squares estimators. Estimation of variance. Confidence intervals and hypothesis testing in multiple regression. Evaluating the appropriateness of the model. Multicollinearity. Selection of variables. Introduction to logistic regression models. Coefficients of the models. Quality of fit. Explore modifying effect and control for confounding in multifactor logistic regression models.

Optional courses in Numerical Analysis and Computational Methods

Tópicos de Teoria da Aproximação/Topics on Approximation Theory [S1 :: 21h :: 3 ECTS]

Resolution of the main problem of approximation theory in metric, normed linear and inner product spaces. Orthogonal bases. Gram-Schmidt algorithm. Convexity. Existence and unicity of best approximations.

Least-squares approximation of empirical and continued functions by algebraic, trigonometric or generalized polynomials. Generalized Fourier series. Orthogonal polynomials of Legendre and Chebyshev. Gaussian quadrature. Formulas of Gauss-Legendre.
Uniform approximation of continued functions by Bernstein polynomials. Simultaneous approximation of a function and its derivatives. Quality of Bernstein approximation. Minimization of the error of polynomial interpolation. Hermite interpolation. Féjer-Hermite uniform approximation.
Some more specific themes in approximation theory (individual projects).

Métodos Espectrais Numéricos/Numerical Spectral Methods [S1 :: 42h :: 6 ECTS]

Introduction to spectral methods. Weighted residual method. Approximation to the function. Differential equations: Galerkin method, Tau method and collocation method. Periodic problems. Fourier transform. Truncated Fourier transform. Discrete truncated Fourier transform (FFT). Elimination of aliasing. Multi-dimensional Fourier transform. Differential equations with constant and variable coefficients. Introduction to evolution equations. Heat equation. Explicit and implicit discretization. Convergence, consistency and stability. Numerical methods for the advection-diffusion equation. Spectral and pseudo-spectral methods. 2- Overview of orthogonal polynomials. Tchebyshev polynomials and main properties. The derivative of order p of Tchebyshev polynomial. A series of truncated Tchebyshev. Derivative matrix. The discrete truncated Tchebyshev serie. The discrete Tchebyshev transform. Differential equations with constant coefficients with boundary conditions. Tchebyshev expansions. The Tau method. Quasi-tridiagonal system of linear equations. The diffusion equation, the Tau method and the Tchebyshev polynomials. Presentation and resolution of many examples in FORTRAN 77, Matlab or Python.

Tópicos de Análise Numérica/Topics in Numerical Analysis [S2 :: 21h :: 3 ECTS]

A selection of topics from the following: Errors and stability: Floating point systems. Errors and their propagation. Stability and conditioning. Interpolation and Approximation: The general problem of interpolation. Interpolation by algebraic polynomials; spline interpolation; trigonometric interpolation. Fourier series. The problem of best approximation. Best approximation in inner product spaces. Least square approximation (continuous and discrete). Minimax approximation. Quadrature: Newton-Cotes rules (simple, composite and adaptative). Peano kernel theorem. Quadrature rules with free knots. Gauss-Christoffel rules. Classical orthogonal polynomials. Gaussian quadrature. Extrapolation methods. Linear Systems: Direct methods for solving linear systems: triangular systems; Gaussian elimination; LU decomposition; Cholesky decomposition. Conditioning. Nonlinear Equations: Iterative methods for solving scalar nonlinear equations. Fixed point iteration. Secant method. Newton's method. Multiple roots. Nonlinear systems: Newton's method and some variants. Differential Equations: Initial value problems. One-step numerical methods. Stability, consistency and convergence analysis. Regions of absolute stability. Multi-step methods. Study of the stability and convergence. Predictor-corrector methods. Boundary value problems. Finite difference methods.