MAP ∞ PDMA
Minho :: Aveiro :: Porto
Programa de Doutoramento em Matemática Aplicada

Study Plan :: 1st year

UC	Semester	ECTS	Remarks
Seminar	S1	9	compulsory
Research Project in Mathematics	S2	21	compulsory
Structural courses	S1	12	choose 2 UC
Optional courses	S1/S2	12	choose 9 ECTS for S1 and 3 ECTS for S2
Any 3rd cycle of studies	S2	6	choose 6 ECTS from any other 3rd cycle or from the Optional courses

The choice of all optional courses is subject to approval by the Scientific Committee of MAP-PDMA.

Syllabi

I - Structural courses :: *Specialised Modules in Mathematics and Applications A*

Tópicos Avançados de Álgebra, Lógica e Computação/ Advanced Topics in Algebra, Logic and Computation [S1 :: 56h :: 6 ECTS]

Each year are taught three of the following topics:
– Computer algebra: introduction to some computer algebra system; development of topics in computational number theory or in computational group theory.
– Automata, languages, and semigroups: regular languages; recognizability by finite state automata and by semigroups; (option 1) varieties of semigroups and languages, Eilenberg's theorem; (option 2) Chomsky's hierarchy, decidability problems.
– Algebraic logic: elements of universal algebra; algebraization of classical, intuitionistic and modal logics; abstract algebraic logic.
– Category theory: universal properties; constructions in categories; natural transformations and adjunctions; monads.
Proof theory: lambda-calculus; intuitionistic logic and Curry-Howard correspondence; proof systems.
– Matrix theory: elementary divisors and invariant factors, minimal polynomial; canonical forms of a matrix; nonnegative matrix, irreducibility and primitivity.

Tópicos Avançados de Análise e Otimização/ Advanced Topics in Analysis and Optimization [S1 :: 56h :: 6 ECTS]

Vector spaces: normed linear spaces; Banach spaces; separability; Lp-spaces. Hahn-Banach Theorem: Open Mapping Theorem; dual spaces; reflexivity; weak and weak-* topologies. Hilbert spaces: the Projection Theorem; Stampacchia e Lax-Milgram Theorems; Riesz Representation Theorem. Application of the Hahn-Banach Theorem to minimum norm problems. Optimization of functionals: Gateaux and Frechet derivatives; Euler-Lagrange equations; problems with constraints; convex-concave functionals; conjugate functionals; dual optimization problems. Global constrained optimization: Lagrange multipliers; sufficiency; sensitivity; duality. Local constrained optimization: Inverse function equality and inequality constraints. Application to optimal control: Pontryagin maximum principle.

Tópicos Avançados de Dinâmica e Geometria/Advanced Topics in Dynamics and Geometry [S1 :: 56h :: 6 ECTS]

Elementary geometry of submanifolds of $\mathbb{R}^n$: Parametrisations (or charts), tangent bundle, differentiable functions, submanifolds, transversality. Differential forms, de Rham cohomology. Basic concepts of dynamics in $\mathbb{R}^n$ (or in submanifolds of $\mathbb{R}^n$): Differential equations, stability of equilibria and of periodic solutions, hyperbolicity, stable and unstable manifolds, Poincaré map. Structural stability and bifurcations. The same concepts for the dynamics of recurrence relations.

Tópicos Avançados de Probabilidade e Estatística/Advanced Topics in Probability and Statistics [S1 :: 56h :: 6 ECTS]

Measurable spaces. Sequences of events. Measurable functions. Measures. Random variables, probability measures, fundamental properties. Probability spaces, types of probability laws. Integration in probability spaces and expectation. Inequalities. Some probability distributions. Independence and conditioning. Characteristic functions. Modes of convergence of sequences of random variables. Laws of large numbers. Central limit theorems. Multivariate distributions, conditional laws. Conditional expectation. Statistical models. Decision theory: risk functions, decision rules, criteria. Exponential families. Sufficiency. Point estimation, comparison of estimators, asymptotic properties, methods of estimation with emphasis on likelihood based inference. Hypothesis tests and confidence sets.

II - Optional courses :: *Specialised Modules in Mathematics and Applications B*

- :: *Probability and Statistic*
- :: *Dynamic and Geometry*
- :: *Numerical Analysis and Computational Methods*
- :: *Control and Optimization*
- :: *Analysis*
- :: *Algebra, Logic and Computation*

Optional courses in Probability and Statistic

Amostragem e Controlo Estatístico da Qualidade/Sampling and Statistical Quality Control [S1/S2 :: 42h :: 6 ECTS]

Sampling

Introduction. Some empirical or non-probabilistic sampling methods: quota sampling; random-route sampling; snowball sampling. Probabilistic sampling methods: simple random sampling; stratified simple random sampling; cluster and multi-stage sampling.

Statistical Quality Control

Introduction. Most important statistical tools in quality control. Acceptance sampling: types of sampling plans. Statistical Process Control: Brief introduction to control charting. Different types of control charts. Sampling politics: FSI (Fixed Sampling Intervals) and VSI (Variable Sampling Intervals). Properties of FSI and VSI control charts. Measures of performance: Type I and type II errors; ARL (Average Run-Length), ANSS (Average Number of Samples to Signal) and ATS (Average Time to Signal). The most common Shewhart control charts for quantitative variables.

Análise Estatística Multivariada/Multivariate Statistical Analysis [S1/S2 :: 42h :: 6 ECTS]

Multivariate random variables. Descriptive statistics. Distances. Generalized variances and algebraic/geometric aspects. Multivariate distributions: Multinormal, Wishart, Hotteling's T2 and Wilks. Inferences about mean vectors; testing for equality of covariance matrices; inferences about mean vectors from two populations. Confidence regions and comparison of multivariate Normal populations. MANOVA -"Multivariate Analysis of Variance". Principal Component Analysis; Factorial analysis. Classification (Cluster Analysis). Discriminant Analysis.

Bioestatística/Biostatistics [S1/S2 :: 42h :: 6 ECTS]

Biological data. Population and sample. Regression models. Linearization of the models. Estimation of regression function. Inferences in regression analysis. Adequacy of the regression model. Coefficient determination. Correlation coefficient. One-way analysis of variance: comparing several means, the analysis of variance F test, conditions for ANOVA, pairwise multiple comparisons. Two-way analysis of variance: conditions, main effects, and interaction. Multiple Regression Model. Multiple Regression Models. Partial coefficient of determination and adjusted coefficient of determination. Properties of least squares estimators. Estimation of variance. Confidence intervals and hypothesis testing in multiple regression. Evaluating the appropriateness of the model.

Multicollinearity. Selection of variables. Introduction to logistic regression models. Coefficients of the models. Quality of fit. Explore modifying effect and control for confounding in multifactor logistic regression models.

Estatística Bayesiana/Bayesian Statistics [S1/S2 :: 42h :: 6 ECTS]

Frequentist Statistics versus Bayesian Statistics: subjective probability; prior information; sampling information; Bayesian methodology and Frequentist methodology; advantages and disadvantages; Bayes theorem as an updating information tool, prior and posterior distributions. Eliciation of prior distributions; non-informative and conjugate distributions. Bayesian inference; estimation, hypothesis testing, prediction and model comparison. Analysis of discrete models: binomial e Poisson models. Inference on the normal model. Simulation methods: Simple Monte Carlo and Markov chain Monte methods. WinBUGGS sofware.

Estatística Genómica/Statistical Genomics [S1/S2 :: 42h :: 6 ECTS]

Genomic data; Analysis of genomic words and words distribution; Algorithms for genome annotation; Comparison of genomic sequences, alignment and free alignment algorithms; Phylogenetic trees, super trees and comparing trees; Analysis of NGS (Next Generation Sequencing) and GWAS (Genome Wide Association Studies); Analysis of other genomic data (e.g. microarrays).

Processamento de Sinal e Séries Temporais/Signal Processing and Time Series Analysis [S1/S2 :: 42h :: 6 ECTS]

Foundations of Statistical Signal Processing. Topics of probabilistic methods in signals and systems, signal joint analysis, modelling, spectral estimation and filtering; application to the analysis of time series in the frequency domain. Selected advanced topics of statistical signal processing, regarding novel methodologies and targeting both longstanding and emergent signal processing and time series applications, as: time-variant and wavelet analysis, adaptation, kernel based learning, independent component analysis, non-linear modeling, bayesian signal processing. State-space models (dynamic linear models) as a general time series modeling framework: ARMAX models, switching models, long-memory models, volatility models, longitudinal data analysis. Case study application and critical insight of the studied methods.

Sistemas Estocásticos/Stochastic Systems [S1/S2 :: 42h :: 6 ECTS]

Module I
Stochastic processes. Markov processes. Markov chains in discrete and continuos time. Wiener process. Stochastic Calculus. Wiener integral. Itô processes and Itô formula. Stochastic differential equations. Linear stochastic differential equations.
Module II
Simulation of stochastic systems in discrete and continuous time. Queuing models. Structure and classification. Key performance measures. Little's Law and fundamental relationships. Process of birth and death. Analysis of systems with one line. Main features. Modeling of systems for which the service time has arbitrary distribution. Formula of Pollaczek-Khinchin. Characterization of the distribution of time spent in the system by using embedded Markov chains. Queues of M/G/1 type with pauses of the server.
Module III
Discretization methods of EDEs. Simulation of time series. Simulation of stochastic differential systems.

Tópicos de Séries Temporais/Topics in Time Series Analysis [S1/S2 :: 42h :: 6 ECTS]

Stationary time series: basics in the univariate and multivariate contexts. Convergence modes and central limit theorems. ARIMA models: causal and invertible ARMA models; forecasting with ARMA models; estimation of ARMA models and properties of estimators; Wold decomposition; integrated models for non-stationary series; ARIMA modelling. Non linera models for time series: introduction; threshold models; bilinear models; GARCH models; INARMA models; Periodic models. Statistical Inference: introduction; estimation, model selection. Periodic AR (PAR) models; Hidden Markov models; functional autoregressive models; spatio-temporal models; time series monitoring using ROC curves, dimensionality reduction: SSA and MSSA.

Análise de Dados Espaciais/Spatial data analysis [S1/S2 :: 21h :: 3 ECTS]

Identify spatially continuous data (geostatistics). Descriptive analysis of spatially distributed data. Types of stationarity (strong, weak or intrisic) and isotropy. Estimation of spatial dependence (variograma and covariogram). Spatial prediction using simple, ordinary or universal kriging. The gaussian geostatistical model and the generalized linear spatial model. Modelling lattice data: spatial association measures; the conditional and the simultaneous auto-regressive models, CAR and SAR (e.g. disease mapping). Brief introduction to spatio-temporal data.

Análise de Dados Longitudinais/Longitudinal Data Analysis [S1/S2 :: 21h :: 3 ECTS]

Linear Mixed Models for continuous data. Marginal models for discrete data. Correlation structures for correlated data. Variograma on longitudinal data. Diagnostics for longitudinal models. Method of maximum likelihood and restricted maximum likelihood. "Missing" data and processes of "Missing" data. Joint modeling of longitudinal and survival data.

Análise e Processamento de Imagem/Image Processing and Analysis [S1/S2 :: 21h :: 3 ECTS]

Fundamentals of digital image processing. Intensity operations and spatial filtering. Colour models. Image segmentation. Morphological operators. Multi-spectral image classification. Geometric correction and image referencing. Image processing in the frequency domain. Representation and object recognition. Examples of Applications.

Classificação de Dados: Metodologias, Validação e Consenso/Data Clustering: Methods, Validation and Consensus [S1/S2 :: 21h :: 3 ECTS]

Main Classification Models and Comparison Measures. Non-Hierarchical Clustering. Hierarchical Clustering. Validation Approaches. Consensus of Classifications.

Complementos de Análise Matemática Aplicada/Complements of Applied Mathematical Analysis [S1/S2 :: 21h :: 3 ECTS]

Introduction/revision of Stochastic Calculus. Model of the Financial Market. Introduction/revision of Numerical Methods and simulation of Stochastic Processes. Computation of European and American Options.

Complementos de Estatística/Complementary Statistics [S1/S2 :: 21h :: 3 ECTS]

Tests of goodness of fit: Chi-square test and Kolmogorov-Smirnov test. Test of Independence. Signal test. Wicoxon test. Location tests for two populations with paired samples. Mann-Whitney-Wilcoxon test. One-way analysis of variance. Methods for multiple comparisons. Kruskal-Wallis test. Two-way analysis of variance. Methods for multiple comparisons.

Estatística Aplicada a Ensaios Clínicos/Statistics Applied to Clinical Trials [S1/S2 :: 21h :: 3 ECTS]

What is a clinical trial? Historical perspective and regulations. Introductory statistical concepts: bias, confounding, clinical significance, reproducibility and generalization. Considerations on clinical trials planning, design and classification. Methodological issues: Randomization and blinding; Data issues and sampling dimension; Censored data and interim analysis. Issues of evaluation: efficacy and safety. Building up and implementing a clinical protocol.

Estatísticas de Ordem e Aplicações/Order Statistics and Applications [S1/S2 :: 21h :: 3 ECTS]

Order statistics and the vector of ranks. Representations and transformations. Role of the order statistics in statistical inference. Limit distributions of central order statistics. Importance of the study of the extremes in diversified domains. Probabilistic characterisation and statistical extreme value modelling. Some of the topics covered are: the weak convergence of extremes and exceedances, the generalized extreme (GEV) and Pareto distributions (GPD), point process methodology, inference in extreme value models. Examples of application, some advanced topics and discussion of problems of research interest in the field.

Extração de Conhecimentos em Fluxos Contínuos de Dados/Data Stream Mining [S1/S2 :: 21h :: 3 ECTS]

Spatio-temporal data analysis. Data streams: main algorithms for classification, clustering, novelty and change detection. Case studies. Social Network Analysis.

Filtragem de Sistemas Parcialmente Observados/Filtering of stochastic systems [S1/S2 :: 21h :: 3 ECTS]

Stochastic models for linear and nonlinear dynamics. Optimal linear filters, predictors and smoothers. The Kalman-Bucy and their properties. Study of the Riccati equation. Non-linear filtering. The Extended Kalman Filter and its properties. Practical aspects. Applications in various areas.

Inferência Não Paramétrica/Nonparametric Inference [S1/S2 :: 21h :: 3 ECTS]

Review of the fundamental concepts and principles of the statistical inference. Nonparametric inference in relation to the parametric one. Nonparametric density estimation. Tests of goodness of fit. Order statistics. Quantiles. Rank-based tests. Measures of association for two variables. Resampling methods, with emphasis to bootstrap. Examples of application, some advanced topics and discussion of specific problems in the field.

Métodos Estatísticos em Epidemiologia/Statistical Methods in Epidemiology [S1/S2 :: 21h :: 3 ECTS]

Epidemiological research: objectives and methods. Typology and design of epidemiologic research. Measures of disease frequency, measures of association and measures of potential impact. Validity of epidemiological research: selection bias, information bias and confounding. Principles and procedures of epidemiologic analysis: options for control of extraneous factors, interaction, effect modification and synergism. Modeling: theoretical considerations and analysis strategy.

Modelos de Dinâmica Estocástica: Simulação e Estimação/Stochastic Dynamics models: simulation and estimation [S1/S2 :: 21h :: 3 ECTS]

Stochastic dynamics in discrete time and stochastic dynamics in continuous time. Connection with the theory of time series. Stochastic differential equations modeling. Simulation. Analysis of solutions. Parameter estimation and estimation of stopping times.

Modelos Lineares Generalizados/Generalized Linear Models [S1/S2 :: 21h :: 3 ECTS]

Review of linear models. Introduction to generalized linear models. Estimation of the model parameters, hypothesis testing and confidence intervals. Selection and validation of models. Regression models for binary data. Regression models for count data. Regression models for asymmetric models.

Regressão Logística e Análise de Sobrevivência/Logistic Regression and Survival Analysis [S1/S2 :: 21h :: 3 ECTS]

Introduction to Logistic Regression. Modelling Strategy Guidelines. Statistical Inferences - Odds Ratio in Logistic Regression. Modelling Strategy for Assessing Interaction and Confounding. Assessing Goodness of Fit for Logistic Regression. Assessing Discriminatory Performance of a Binary Logistic Model: ROC Curves. Introduction to Survival Analysis. Kaplan-Meier Survival Curves and the Log-Rank Test. The Cox Proportional Hazards Model and Its Characteristics Evaluating the Proportional Hazards Assumption. Extension of the Cox Proportional Hazards Model for Time-Dependent Variables Competing Risks Survival Analysis.

Sistemas com acontecimentos discretos/Discrete event system [S1/S2 :: 21h :: 3 ECTS]

Introduction. Untimed Models of Discrete-Event Systems: languages and automata theory, analysis of untimed models. Timed Models of Discrete-Event Systems: timed state automata. Stochastic Timed Models for Discrete-Event Systems: introduction to stochastic processes, stochastic timed state automata, generalized semi-Markov process, Poisson process. Markov Chains: models, transition probability matrix, transient and steady state analysis for discrete-time Markov chains and continuous-time Markov. Controlled Markov Chains: Markov decision processes, approaches to the synthesis for Markov decision problems. Queueing Theory: queueing models, performance and dynamics of a queuing system, analysis of Markovian queueing systems, Markovian queueing networks, control of queueing systems. Discrete-Event Simulation: the event scheduling simulation scheme, the process-oriented simulation scheme, discrete-event simulation languages, output analysis. Sensitivity Analysis.

Tópicos em Genética Estatística/Topics in Statistical Genetics [S1/S2 :: 21h :: 3 ECTS]

Mendelian inheritance. Modes of transmission and risks calculation. Epidemiologic principles. Measures of disease frequency and association. Familial risk of disease. Segregation analysis. Linkage analysis: recombination fraction, genetic distance, haplotype. Lodscore methods. Population genetics: Hardy-Weinberg equilibrium. Linkage desequilibrium. Association studies.

Optional courses in Dynamic and Geometry

Alterações Climáticas e Energia/Climate change and energy [S1/S2 :: 42h :: 6 ECTS]

Dynamics and economical models in Game Theory, General equilibrium theory and Mathematical Finance, applied to climate change and energy models.

Dinâmica em Modelos Biológicos/Biological Dynamics [S1/S2 :: 42h :: 6 ECTS]

Dynamic models in Ecology, Epidemiology, Immunology and in Biological systems. Introduction to Dynamical equilibria, stability, Lyapunov functions, bifurcations, topological chaotic dynamics. Stochastic stability, quasistationary equilibria. Nash equilibria, Stochastically stable equilibrium, evolutionarily stable state, evolutionary game theory. SIS, SIR, SIRI models: thresholds, bifurcations, mean field approximation, pair approximation, quasistationary equilibria, vaccination, game theoretical effects in vaccination. T-cells and regulatory T cells dynamics modeling: immune deceases, bifurcation thresholds, limit cycles, data analysis. Systems biology and dynamic analysis. Bioenergy models.

Equações diferenciais com simetria/Differential equations with symmetry [S1/S2 :: 42h :: 6 ECTS]

Qualitative study of differential and difference equations, parameter dependence, bifurcation and stability. Study of symmetry breaking in steady-state and Hopf bifurcations. The aim is to show how symmetries may be used systematically to analyze, predict, and understand dynamic behaviour. Tools to be used are Lie groups; representations and actions; invariant and equivariant maps, Hilbert-Weyl theorem.

Métodos Avançados em Teoria da Relatividade/Advanced Methods in Relativity Theory [S1/S2 :: 42h :: 6 ECTS]

Review of the main results and geometrical structure of Special Relativity. Foundations and main results of Differential Geometry. Metric connexion and covariant derivative. Manifold curvature and associated tensors. Parallel transport. Geodesic equations. The relation between curvature and gravitation. The Equivalence Principle. The Einstein field equations. Einstein Equations solutions. The perihelion shift effect. Light deflection in the presence of strong gravitational fields. Interior and exterior Schwarzchild solutions. Cosmological solutions. Gravitational waves.

Teoria de Bifurcação/Bifurcation Theory [S1/S2 :: 42h :: 6 ECTS]

Introduction to the study of the alteration of the qualitative behavior of dynamical systems with parameters. These alterations include, for instance, the appearing and disappearing of equilibrium states, changes in the periodic behavior of solutions, changes on the stability or lead to a chaotic behavior.

Geometria Combinatória/Combinatorial Geometry [S1/S2 :: 21h :: 3 ECTS]

Maps and hypermaps. Actions of discrete groups. Riemann Surfaces. Euclidean and non-Euclidean crystallographic groups. Fuchsian Groups. Algebraic curves. Topological graphs.

Geometria Lorentziana/Lorentzian Geometry [S1/S2 :: 21h :: 3 ECTS]

Lorentzian manifolds. Causality theory, isometry groups. Minkowski Geometry. Completeness and extendability. Singularities. Hyperbolicity.

Geometria Riemanniana/Riemannian Geometry [S1/S2 :: 21h :: 3 ECTS]

Riemannian metrics. Levi-Civita connection. Geodesics. Normal neighborhoods. Curvature Tensor. Covariant derivation of tensors. Jacobi fields and conjugated points. Isometric immersions; Gauss, Ricci and Codazzi equations. Complete Riemannian manifolds; Hopf-Rinow theorem, Hadamard's theorem.

Mecânica Geométrica/Geometrical Mechanics [S1/S2 :: 21h :: 3 ECTS]

Symplectic Geometry: symplectic geometry in vector spaces and in differential manifolds. Hamilton mechanics: study of mechanical systems from the Hamiltonian point of view. Examples. Symplectic actions of Lie groups, momentum and reduction: the use of symmetries of a mechanical system in reduction to lower dimension. Examples.

Métodos Geométricos Avançados da Física/Advanced Geometrical Methods of Physics [S1/S2 :: 21h :: 3 ECTS]

Differentiable manifolds, fiber bundles, differential forms, exterior calculus, tensors, Lie derivative, Frobenius theorem, notions of Lie groups and algebras, the exponential map, unitary group, orthogonal group, decompositions. Applications to Physics.

Relatividade Matemática/Mathematical Relativity [S1/S2 :: 21h :: 3 ECTS]

Conformal compactification of spacetime and Penrose diagrams, causality and global hyperbolicity. Singularity theorems of Hawking and Penrose. 3+1 decomposition and the Cauchy problem for the Einstein equations. Choquet-Bruhat theorem. Perturbations and stability.

Teoria de Singularidades I/Singularity Theory I [S1/S2 :: 21h :: 3 ECTS]

Curves and functions on them. Regular values and smooth manifolds. Envelopes. Unfoldings.

Teoria de Singularidades II/Singularity Theory II [S1/S2 :: 21h :: 3 ECTS]

Transversality. Generic properties of curves. Singular points, several variables and generic surfaces.

Tópicos de Geometria Diferencial/Topics of Differential Geometry [S1/S2 :: 21h :: 3 ECTS]

Spinor algebra, The Petrov classification, spinor analysis, Newmann-Penrose formalism, Golberg-Sachs Theorem. Conformal transformations, Conformal infinity. Asymptotics in Minkowski space. Conformally invariant operators. Conformal Einstein equations: the characteristic initial value problem; the vacuum case.

Topologia Algébrica/Algebraic Topology [S1/S2 :: 21h :: 3 ECTS]

Homotopy. Fibrations and cofibrations. CW-complexes. Homology. Cohomology. Homotopy groups.

Optional courses in Numerical Analysis and Computational Methods

Álgebra Linear Numérica/Numerical Linear Algebra [S1/S2 :: 42h :: 6 ECTS]

Errors in matrix computations. Error analysis in basic matrix computations. Backward error analysis. Conditioning of linear systems and of eigenvalues and eigenvectors. Block algorithms. Permutation matrices, Gaussian elimination matrices, Givens' matrices and Householder's matrices. LU, QR and Choleski decompositions. Block versions of the decompositions. The routines from BLAS. Systems of linear equations Sparse matrices storage. Iterative methods for sparse systems. The basic methods of Jacobi, Gauss-Seidel and SOR. Krylov-type methods. Preconditioning techniques. Eigenvalues and eigenvectores. Orthogonal similarity reductions to Hessenberg and tridiagonal forms. The QR method. The bisection method for symmetric tridiagonal matrices. The power method and inverse iteration. Lanczos' and Arnoldi methods Singular values. The singular value decomposition (SVD). The Golub-Kahan and Lanczos methods for the reduction to bidiagonal form. QR-type and bisection methods for the bidiagonal SVD problem.

Complementos de Análise Númerica I/Numerical Analysis I [S1/S2 :: 21h :: 3 ECTS]

Introduction to numerical analysis. Systems of linear equations: Gaussian elimination. Systems of Non-linear equations: Newton's method and variants. Symmetric eigenvalue problem: QR algorithm, inverse iteration, Rayleigh quotient iteration. Polynomial interpolation: Lagrange, Hermite. Numerical integration: Newton-cotes, Gaussian quadrature. Initial value problems for (systems of) ordinary differential equations: consistency and convergence, Runge-Kutta. Boundary value problems for ordinary differential equations: shooting and finite differences.

Complementos de Análise Númerica II/Numerical Analysis II [S1/S2 :: 21h :: 3 ECTS]

Introduction: numerical linear algebra problems, finite precision arithmetic, matrix factorizations, conditioning and stability. Algorithms for high performance computing: computer architecture, data structures. Systems of linear equations: stationary and non-stationary iterative methods. Preconditioners. Eigenvalue problems: symmetric and non-symmetric.

Métodos Computacionais em Hidrodinâmica Costeira/Computational Methods in Coastal Hydrodynamics [S1/S2 :: 21h :: 3 ECTS]

Characteristics of coastal waters. Short surface waves. Long surface waves. Wave modulation and breaking. Wind-induced wave statistics and spectral properties. Currents in coastal areas: wave-induced currents; tidal currents; wave-current interaction. Computational methods for discretization of wave and circulation models. Monochromatic waves. Eliptic Helmholtz and Berkhoff equations. Discretization by the finite element method(FEM). Phase resolving models (dispersive long wave models, Boussinesq). Discretization of long waves models by the FEM. Phase averaging models. Spectral description of waves. Discretization in the space of phase. Circulation models. FEM for the Saint-Venant equation.

Métodos de Elementos Finitos para Equações de Ondas Dispersivas/Finite Element Methods for Dispersive Wave Equations [S1/S2 :: 21h :: 3 ECTS]

Models for kinematic water waves: the KdV equation; the BBM equation; a KdV-BBM improved dispersion equation. Models for dynamic waves: the Boussinesq equations; improved dispersion relation Boussinesq-type equations. Finite Element Methods: Galerkin method; Petrov-Galerkin method; Taylor-Galerkin method; Discontinuous Garlerkin method. Programming and stability and accuracy analysis.

Métodos Espectrais Numéricos I/Numerical Spectral Methods I [S1/S2 :: 21h :: 3 ECTS]

Introduction to spectral methods. Weighted residual method. Approximation to the function. Differential equations: Galerkin method, Tau method and collocation method. Periodic problems. Fourier transform. Truncated Fourier transform. Discrete truncated Fourier transform (FFT). Elimination of aliasing. Multi-dimensional Fourier transform. Differential equations with constant and variable coefficients. Introduction to evolution equations. Heat equation. Explicit and implicit discretization. Convergence, consistency and stability. Numerical methods for the advection-diffusion equation. Spectral and pseudo-spectral methods. Presentation and resolution of many examples in FORTRAN 77, Matlab or Python.

Métodos Espectrais Numéricos II/Numerical Spectral Methods II [S1/S2 :: 21h :: 3 ECTS]

Overview of orthogonal polynomials. Tchebyshev polynomials and main properties. The derivative of order p of Tchebyshev polynomial. A series of truncated Tchebyshev. Derivative matrix. The discrete truncated Tchebyshev serie. The discre te Tchebyshev transform. Differential equations with constant coefficients with boundary conditions. Tchebyshev expansions. The Tau method. Quasi-tridiagonal system of linear equations. The diffusion equation, the Tau method and the Tchebyshev polynomials. Presentation and resolution of many examples in FORTRAN 77, Matlab or Python.

Métodos Numéricos para Equações Diferenciais/Numerical Methods for Differential Equations [S1/S2 :: 21h :: 3 ECTS]

Initial value problems. Existence theorem. One-step numerical methods. Euler and Runge-Kutta methods: stability, consistency and convergence analysis. Regions of absolute stability. Multi-step methods. Study of the satbility and convergence. Taylor methods. Predictor-corrector methods. Boundary value problems. Finite difference methods.

Tópicos de Análise Numérica/Topics in Numerical Analysis [S1/S2 :: 21h :: 3 ECTS]

Errors and stability. Floating point systems. Errors and their propagation. Stability and conditioning. Interpolation and Approximation. The general problem of interpolation. Lagrange and Hermite polynomial interpolation. Splines. Trigonometric interpolation. Fourier series. The problem of best approximation. Best approximation in inner product spaces. Least square approximation. Minimax approximation. Quadrature Newton-Cotes rules (simple, composite and adaptative). Peano kernel theorem. Gauss-Christoffel rules. Classical orthogonal polynomials. Gaussian quadrature. Extrapolation methods. Linear Systems Direct methods for solving linear systems: triangular systems; Gaussian elimination; LU decomposition; Cholesky decomposition. Conditioning. Nonlinear equations. Iterative methods for solving scalar nonlinear equations. Fixed point iteration. Secant method. Newton's method. Multiple roots. Nonlinear systems: Newton's method and some variants.

Optional courses in Control and Optimization

Otimização e Desenho de Redes/Optimization and Network Design [S1/S2 :: 42h :: 6 ECTS]

Models and algorithms used to solve (combinatorial) optimization problems that appear in network (telecommunications, transport, etc.) design problems. Basic concepts: flows, cuts, paths, etc. Discussion of formulations. Solution techniques: relaxations, cutting planes and heuristics. Network design with high level of service: availability, reliability, survivability.

Otimização Estratégica em Economia/Strategic optimization in Economy [S1/S2 :: 42h :: 6 ECTS]

General equilibria theory. Game theory. Applications to energy and climate change.

Sistemas de Apoio à Decisão/Decision Support Systems [S1/S2 :: 42h :: 6 ECTS]

Introduction to Decision Support Systems (DSS): historical perspective; operational systems vs. analytical systems; data Warehouse; DSS architecture; DSS components. Some decision support methods: decision analysis; soft systems methodology. Combinatorial optimization: introduction to combinatorial optimization; the Travelling Salesman Problem (TSP) and the Vehicle Routing Problem (VRP); Cutting-Stock Problem; Vehicle Routing and Container Loading Problem; Location Analysis. Decision support tools.

Análise e Controlo de Sistemas Lineares/Analysis and Control of Linear Systems [S1/S2 :: 21h :: 3 ECTS]

State spaces models of linear multivariable systems, solution in the frequency domain and in the time domain (transfer function and impulse response). State spaces models of linear systems as linearization of nonlinear

systems. Structural properties. Elements of the theory of realization. Systems interconnection. Control problems with state measurement: pole placement and stabilization by static feedback; tracking and disturbance rejection. Control problems with state estimation. A deterministic version of the Kalman filter.

Bilhares, problemas de resistência ótima e transporte de massa ótimo/Billiards, problems of optimal resistance and optimal mass transportation [S1/S2 :: 21h :: 3 ECTS]

Definition and main properties of billiards. Billiards in polygons and domains bounded by curves of 2nd order. Unfolding of a billiard trajectory. Problems of minimal resistance: discussion, definition, main properties, and basic theory. New theory of minimal resistance of convex and nonconvex bodies under the single impact condition. Billiard approach to the problems of minimal resistance. Minimal resistance of rotating bodies and optimal mass transportation. Basic notions of the optimal mass transport theory; Monge problem, Kantorvich relaxation, cyclic monotonicity. The main theorem of minimum mean resistance and a sketch of the proof.

Cálculo das Variações/Calculus of Variations [S1/S2 :: 21h :: 3 ECTS]

We address the Calculus of Variations from an integrated point of view. Example of topics to be studied: equivalence between different formulations of the problems; existence of minimizers; necessary optimality conditions; sufficient conditions for optimality; applications.

Códigos e Sistemas/Codes and Systems [S1/S2 :: 21h :: 3 ECTS]

Introduction to the theory of polynomial matrices: unimodular matrices, right prime matrices and column reduced matrices. Convolutional code over a finite field. Encoders of a convolutional code: definition and properties. Realization of an encoder through a state space model: definition and an algorithm to construct a minimal realization of an encoder. Structure of the minimal encoders.

Complementos de Teoria do Controlo/Complements of Control Theory [S1/S2 :: 21h :: 3 ECTS]

Selected topics in:
– Optimal control: nondegeneration, sensitivity, regularity, impulsive control, etc.
– Nonlinear control: geometric control, stability, synthesis of nonlinear controllers, invariance, atingibility, multi-dimensional systems, etc.
– Model predictive control: stability, robustness, complexity, sampling strategies, etc.
– Coordinated control: formulation of the diverse types of problems, control synthesis methods, optimization, and robustness.
– Decentralized control: formulation of the diverse types of problems, control synthesis methods, consensus generation, optimization, and robustness.
– Robust control: formulation of the diverse types of problems, control synthesis methods.
– Adaptive control: formulation of the diverse adaptive control schemes, control synthesis methods, multi-model approaches, and robust adaptivity.
– Hybrid control systems: stability, control synthesis, invariance, atainability, and optimization.

Controlo Não Linear/Nonlinear Control [S1/S2 :: 21h :: 3 ECTS]

Introduction to nonlinear systems. Mathematical preliminaries. Fundamental properties of nonlinear system. Lyapunov stability. Input-Output stability. Advanced stability. Stability of perturbed systems. Nonlinear design tools.

Controlo Ótimo/Optimal Control [S1/S2 :: 21h :: 3 ECTS]

Optimal control problems. Examples. Formulation. Existence of solution. Necessary conditions. Pontryagin maximum principle. Study of standard problems: linear case, geometric interpretation; Minimum time; LQR problem. Linear and nonlinear problems. Examples. Sufficient optimality conditions.

Equações Dinâmicas em Time Scales/Dynamic Equations on Time Scales [S1/S2 :: 21h :: 3 ECTS]

Time scales calculus
Basic definitions. The delta derivative. Delta differentiation properties. The delta integral. Delta integration properties. The nabla derivative and the nabla integral. Relations between delta and nabla operators. The exponential function on time scales. Dynamics equations on time scales.
Application: Calculus of variations on time scales
Variational problems on time scales. Necessary condition for optimality: Euler-Lagrange equation. Transversality conditions. Sufficient conditions for optimality.

Identificação de Sistemas/System Identification [S1/S2 :: 21h :: 3 ECTS]

– Models of discrete linear time invariant systems:
Deterministic models: (i) input-output models; (ii) state-space models. Stochastic models: (i) input-output models: predictiors; moving average models; auto-regressive models; auto-regressive moving average models; (ii) state-space models: process and measurement noise; Kalman predictor and Kalman filter; prediction error model.
– Identification of input-output models:
Least squares method. Impulse response identification: correlation methods. Instrumental variable methods. Prediction error methods.
– Identification of state-space models. Deterministic realization theory. Subspace methods. Prediction error methods.
– Identification experiment design.
– Introduction to continuous-system identification.

Identificação e Controlo Preditivo de Sistemas/System Identification and Predictive Control [S1/S2 :: 21h :: 3 ECTS]

Identification of input-output models: least squares method; impulse response identification: correlation methods; instrumental Variable methods; prediction Error methods. Identification of state-space models: deterministic realization theory; introduction to Subspace methods. Linear, quadratic optimal control: unconstrained Linear quadratic problems; dynamic programming solution; finite and infinite horizon problems; Controllability and stability; constrained LQ problems; Model Predictive Control: the receding horizon strategy; stability of MPC schemes. Moving Horizon Estimation: linear, Gaussian and unconstrained problems; limitations of the Kalman Filter; MHE as least squares problem; observability and convergence. Output feedback model predictive control: examples; stability with observers.

Métodos de Pesquisa Evolucionária Aplicados a Problemas de Engenharia/Evolutionary search methods applied to engineering problems [S1/S2 :: 21h :: 3 ECTS]

Framework of the discipline in the context of applied mathematics. Review of some fundamental concepts in optimization. Unconstrained optimization problems. Optimization problems with constraints. Introduction to the language and paradigms associated with Evolutionary Search Methods. Genetic Algorithms: definition of the key aspects. Taxonomic analysis of the main genetic operators and mechanisms of evolution of populations. Elitism. Memetic algorithms: genetic and cultural evolution. Algorithms based on the particles intelligence (Particle Swarm Optimization Method): main features. Optimization emulating the behavior of ant colony: fundamentals. Brief reference to other methods of evolutionary search. Applications of search evolutionary methods to optimal d esign of engineering problems: applications in structural engineering, mechanical engineering and manufacturing processes. Performance analysis of different methods of evolutionary search in engineering.

Programação Linear Inteira/Integer Linear Programming [S1/S2 :: 21h :: 3 ECTS]

Basic concepts: formulation, relaxation (linear and lagrangian), upper and lower bounds. Complexity. Polynomialy solvable problems: properties and examples. Valid inequalities and separation. Exact algorithms: Branch and Bound, Branch and Cut. Heuristic algorithms.

Sistemas Comportamentais / Behavioral Systems [S1/S2 :: 21h :: 3 ECTS]

Discrete-time behavioral systems; Linear, time-invariant and complete behaviors (kernel, hybrid and image representations); Fundamental properties (controllability, autonomy, controllable-autonomous decomposition, observability, stability, structural indices); State/driving-var iable representations Input/output representations; Input/state/output representations; Behavioral control; Behavioral observers.

Sistemas Híbridos/Hybrid Systems [S1/S2 :: 21h :: 3 ECTS]

Introduction and motivation. State machines and their composition. Basic concepts and results of linear, nonlinear and switched systems. Hybrid systems modeling: articulation of event-driven and time-driven dynamics; classes of models; timed automata; general formal model; simulation of hybrid systems; mini-project on hybrid systems modeling using MATLAB and StateFlow. Control design: supervisory control; reachability and safety problems; behaviors and requirements specification; control synthesis techniques; exercises and mini-project on hybrid systems control synthesis using the MALAB toolbox.

Teoria de Otimização/Optimisation Theory [S1/S2 :: 21h :: 3 ECTS]

Basic concepts: Elements of convex analysis, convex sets and convex functions, settings, properties and examples. Classification of problems: with and without constraints, convex and non convex problems, specific problems.

Special cases of constraints in the form of equality and inequality, infinite constraints. Unconstrained optimization: optimality conditions, the notion of optimization algorithm and the properties of algorithms. Optimization methods without constraints and without calculus of derivatives. Constrained optimization: classification of problems, optimality conditions I and II order. Classification of optimization methods with constraints: methods of convex and linear programming, barrier functions and penalty interior point methods, methods based on Lagrange multipliers. Problems with linear constraints. Introduction to Data Mining: Optimization methods in solving the problems of classification and clustering.

Optional courses in Analysis

Análise de Clifford e Aplicações/Clifford Analysis and Applications [S1/S2 :: 42h :: 6 ECTS]

Realization of algebra of endomorphisms as Clifford algebra with signature (n, n); construction of Dirac operator; construction of Super-Lie algebra osp(1|2); reduction to the Heisenberg algebra. Differential forms and integral formulae; construction of spherical harmonics and monogenics. Monogenic signals: connection between Riesz transforms and classic Radon transform; generalization to the case of the circular Radon transform and application to curvature detection in images.

Análise Harmónica em Espaço de Fase/Harmonic Analysis in Phase-space [S1/S2 :: 42h :: 6 ECTS]

The Heisenberg algebra and the phase space; raising and lowering operators; Schroedinger representation. Weyl calculus and Weyl algebra; position and momentum operators; Fock space representation; relation with Umbral calculus. Time-frequency analysis and pseudo-differential operators. Short-time Fourier transform for recovering frequency and phase information of local sections of given signals.

Equações com Derivadas Parciais/Partial Differential Equations [S1/S2 :: 42h :: 6 ECTS]

Hölder spaces. Distributions. Sobolev spaces. Traces and imbeddings. Extension theorems. Second order partial differential elliptic equations: existence of weak solutions and regularity. Maximum principle and Harnack inequality. Schauder estimates. Second order parabolic equations: existence of weak solutions and regularity. Maximum principle and Harnack inequality. Monotone parabolic equations.

Frames e Representações Esparsas de Sinais/Frames and Sparse Representations of Signals [S1/S2 :: 42h :: 6 ECTS]

Basic theory of frames (definition, its relation with Riesz basis, properties of the frame operator), discrete and continuous frames, study of Feichtinger/Gröchenig theory, Gabor and wavelets frames, frame dictionaries, Applications to sparse signal representation (variational formulations which promote sparsity, best N-term approximation, Jackson- and Bernstein-type theorems and constructive methods for approximation).

Métodos de Riemann-Hilbert para Polinómios Ortogonais/Riemann-Hilbert method for Orthogonal Polynomials [S1/S2 :: 21h :: 3 ECTS]

Elements of the General Theory of Orthogonal Polynomials. Boundary Value Problems in the Theory of Analytic Functions. Riemann-Hilbert Method for Orthogonal Polynomials on $[-1, 1]$. Non-Linear Steepest Descent Method. Outer and Local Parametrix. Asymptotic results for Ortho gonal Polynomials on $[-1, 1]$.

Problemas Inversos/Inverse Problems [S1/S2 :: 42h :: 6 ECTS]

Concept of well- and ill-posed problems, linear and non-linear regularization methods and corresponding variational formulations. Regularization methods. Radon-Funk transforms and integral geometry on symmetric spaces. Study of numerical methods based on frames with and without adaptivity and their applications to integral geometry.

Teoria Cinética I/Kinetic Theory [S1/S2 :: 42h :: 6 ECTS]

Basic principles: velocity distribution function; collisional dynamics; collisional invariants; Boltzmann equation; collisional operator. Macroscopic description: macroscopic variables; transport equation; balane equations; conservation laws. Equilibrium: equilibrium states; Maxwellian distribution function; Boltzmann H-theorem; entropy. Linearized Boltzmann equation. Properties. Homogeneous Boltzmann equation. Properties. Solution of the Boltzmann equation: existence and uniqueness results; asymptotic behavior of the solution. Solution methods of the solution of the Boltzmann equation: Chapman-Enskog method; transport coefficients and transport fluxes.

Teoria dos Espaços de Funções/Theory of Function Spaces [S1/S2 :: 42h :: 6 ECTS]

Several essential topics in the theory of function spaces of Besov and Triebel-Lizorkin type, both in a more classic context (Fourier analysis approach) and in a broaden one (characterization via atoms, molecules and wavelets;

extension to fractal domains) are presented. Relevant properties for their use in the context of various operators and of the so-called fractal analysis are discussed.

Tópicos de Análise Harmónica/Topics of Harmonic Analysis [S1/S2 :: 42h :: 6 ECTS]

Preliminaries (Lebesgue measure and Lebesgue integral, Lebesgue spaces, distributions). Approximations to the identity (via convolution). Fourier transform. Fourier series. Maximal operators. Fractional integral operators. Hardy-Littlewood-Sobolev Theorem. Singular operators. Basics on interpolation theory.

Análise Não Diferenciável/Nondifferentialble Analysis [S1/S2 :: 21h :: 3 ECTS]

The need for nondifferentiable calculus. History of nondifferentiable calculus. Brief introduction to set valued analysis. Convex analysis. Proximal normals and proximal subdifferentials. Calculus. Limiting normals and subdifferentials. Clarke subdifferentials. Applications to optimization. Applications to optimal control.

Classes de Operadores/Classes of Operators [S1/S2 :: 21h :: 3 ECTS]

General notions about linear and non-linear operators. Wiener-Hopf, Hankel, Toeplitz, singular integral and pseudodifferential operators. Methods for the characterization of the regularity properties of these operators. Factorization methods for classes of operators. Regularizers identification for classes of operators. Representation of (lateral, bilateral and generalized) inverses of operators.

Equações Diferenciais/Differential Equations [S1/S2 :: 21h :: 3 ECTS]

General Theory: vector fields, flow, rectification, existence and uniqueness of solution, first integrals, phase curves, conservative systems. Linear Equations: fundamental solution and stability. Non-Linear Equations: stability, local and global analysis.

Equações Integrais e suas Generalizações/Integral Equations and their Generalizations [S1/S2 :: 21h :: 3 ECTS]

Linear and non-linear integral equations. Integral equations of Fredholm, Volterra, singular, convolution type, with shift and of Hammerstein. Integro-differential equations. Fixed point theory for integral equations. Variational methods for integral equations. Abstract integral equations. Use of integral equations in wave diffraction problems modelling.

Inequações Variacionais e Quasi-Variacionais/Variational and quasi-variational inequalities [S1/S2 :: 21h :: 3 ECTS]

Projection theorem and Lions-Stampacchia theorem. The obstacle problem and Lewy-Stampacchia inequalities. Mosco convergence of convex sets. Fixed point theorems. Penalization methods. Quasi-variational inequalities.

Introdução à Dinâmica Simbólica/Introduction to Symbolic Dynamics [S1/S2 :: 21h :: 3 ECTS]

Elements of the theory of discrete dynamical systems. Symbolic dynamical systems and Markov matrices. Milnor and Thurston kneading theory for modal maps of the interval. Symbolic dynamics for unimodal maps of the interval. Symbolic dynamics for bimodal maps of the interval. Symbolic dynamics for Lozi maps.

Métodos da Teoria de Operadores em Problemas de Valores de Fronteira/Methods of the Theory of Operators in Boundary Value Problems [S1/S2 :: 21h :: 3 ECTS]

Abstract boundary value problems, coercivity, Green formulas for various boundary value problems (in Sobolev type spaces), solutions representation by using layer potentials, generalized potentials traces, weak solution formulation, variational methods, characterization of the solution spaces. Application to wave diffraction problems in homogeneous and isotropic; localization principle; generalized inversion; singularities and asymptotic analysis; normalization.

Teoria Algébrica dos Polinómios Ortogonais e Aplicações/Algebraic Theory of Orthogonal Polynomials and Applications [S1/S2 :: 21h :: 3 ECTS]

– Algebraic theory of orthogonal polynomials: operations in the dual space of polynomials, properties, sequences of polynomials, dual sequences, regular orthogonality: definition and recurrence relation. Perturbed and associated polynomials. Appell polynomials.
– Classical orthogonal polynomials: definition, characterization by the functional equation, by the differential equation, by the structure relations. Invariance of the classic character. Classical forms: four equivalence classes (Hermite, Laguerre, Bessel, Jacobi). Integral representations. Properties of derivation. Properties of zeros.
– Chebyshev polynomials: properties and applications in Numerical Analysis.

– Some applications of orthogonal polynomials: orthogonal series, Padé rational approximation, Gaussian quadrature.
– Some more specific themes in orthogonality (individual projects).

Teoria Cinética II/Kinetic Theory II [S1/S2 :: 21h :: 3 ECTS]

Boltzmann equation for mixtures. Boltzmann equation for chemically reacting gases. BGK (Bhatnagar-Gross-Krook) model. Problems of fluid dynamics: propagation of discontinuities; sound waves; shock-wave structure; steady detonation wave solutions; linear stability of steady detonation. Modelling, numerical analysis and simulation of these problems.

Tópicos de Teoria da Aproximação/Topics on Approximation Theory [S1/S2 :: 21h :: 3 ECTS]

Resolution of the main problem of approximation theory in metric, normed linear and inner product spaces. Orthogonal bases. Gram-Schmidt algorithm. Convexity. Existence and unicity of best approximations.
Least-squares approximation of empirical and continued functions by algebraic, trigonometric or generalized polynomials. Generalized Fourier series. Orthogonal polynomials of Legendre and Chebyshev. Gaussian quadrature. Formulas of Gauss-Legendre.
Uniform approximation of continued functions by Bernstein polynomials. Simultaneous approximation of a function and its derivatives. Quality of Bernstein approximation. Minimization of the error of polynomial interpolation. Hermite interpolation. Féjer-Hermite uniform approximation.
Some more specific themes in approximation theory (individual projects).

Tópicos de Turbulência/Topics in Turbulence [S1/S2 :: 21h :: 3 ECTS]

Navier-Stokes equations. Vorticity dynamics. Statistical description of turbulent flows. Coherent structures. Isotropic turbulence (spectral formalism, Kolmogorov theory). Anisotropic turbulence (SO(3) formalism). Discussion on the existence and uniqueness of solutions of the Navier-Stokes equations. Eddy viscosities. Point vortices. Passive scalars. Direct numerical simulation.

Optional courses in Algebra, Logic and Computation

Teoria de Tipos/Type Theory [S1/S2 :: 42h :: 6 ECTS]

Simple types, polymorphic types, dependent types: meta-theory, propositions-as types principle, relationship with constructive logics. Optional topics: inductive definitions and termination of recursive functions in Type Theory; proof assistants; type systems based on sequent calculus or Classical Logic; subtyping; etc.

Teorias de Galois/Galois Theories [S1/S2 :: 42h :: 6 ECTS]

Classical Galois theory, Galois theory of Grothendieck, Infinitary Galois theory, Categorical Galois theory of commutative rings, Categorical Galois theorem and factorization systems, Covering maps.

Álgebras de Jordan Euclidianas e Grafos/Euclidian Jordan Algebras and Graphs [S1/S2 :: 21h :: 3 ECTS]

Power-associative algebras. Jordan algebras. Euclidian Jordan algebras. Classifications of Euclidean Jordan algebras. Graphs. Euclidian Jordan algebras and strongly regular graphs. Admissibility conditions on the parameters of a strongly regular graph on the environment of Euclidian Jordan algebras.

Elementos da Teoria de Semigrupos/Elements of Semigroup Theory [S1/S2 :: 21h :: 3 ECTS]

Fundamental aspects of semigroup theory. Regular semigroups, inverse semigroups and completely simple semigroups.

Fundamentos da Teoria de Semigrupos Finitos/Fundamental aspects of finite semigroup theory [S1/S2 :: 21h :: 3 ECTS]

Pseudovarieties of finite semigroups; Implicit operations; Pseudovarieties defined by pseudoidentities; Pseudovariety operators; Decidability problems.

Reticulados e Estruturas Algébricas Ordenadas/Lattices and Ordered Algebraic Structures [S1/S2 :: 21h :: 3 ECTS]

Ordered Semigroups: the ordered monoid of residuated mappings on an ordered set; naturally ordered semigroups; ordered regular semigroups; ordered inverse semigroups; representation of some classes of ordered semigoups.

Lattices: distributive lattices; complemented lattices, Boolean algebras; pseudocomplemented lattices, Stone algebras.

Teoria Algébrica das Inversas Generalizadas/Algebraic theory of generalized inverses
[S1/S2 :: 21h :: 3 ECTS]

The concept of generalized inverse, families of generalized inverses, generalized invertibility of matrices over certain algebraic structures, in rings and some of its subclasses, and in C*-algebras

Tópicos da Teoria de Semigrupos de Transformações/Topics of Transformation Semigroups Theory
[S1/S2 :: 21h :: 3 ECTS]

Transformations and semigroups. The semigroups $T(X)$, $P(X)$ and $I(X)$. Generators. Regular elements. Green's relations and ideals. Subsemigroups. Congruences. Endomorphisms.

Tópicos de Semigrupos e Linguagens/Topics of Semigroups and Languages [S1/S2 :: 21h :: 3 ECTS]

Semigroups. Semigroup homomorphisms. Green's relations on semigroups. Automata and languages. Language recognition by automata and by semigroups. Kleene's theorem. Varieties of semigroups and varieties of languages. Eilenberg's theorem.